Velocity of Transverse Domain Wall Motion Along Thin, Narrow Strips

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Abstract— **Micromagnetic simulation of domain wall motion in thin, narrow strips leads to a simplified analytical model. The model accurately predicts the same domain wall velocity as full micromagnetic calculations, including dependence on strip width, thickness, and magnitude of applied field pulse. Domain wall momentum and retrograde domain wall motion are both observed and explained by the analytical model.**

I. Introduction

The dynamics of magnetization reversal are key to the functioning of many nanoscale magnetic devices. The effect of the shape and small dimensions of magnetic elements on magnetodynamics are important to understand, so that devices can be designed to provide controlled reversal and can be optimized to meet reversal speed requirements. In this study we first use micromagnetic simulation to examine domain wall motion in thin, narrow strips of magnetic material. Inspired by the simulation results, we then produce a simpler analytical model that agrees with the full micromagnetic simulation remarkably well. Both the full micromagnetic simulation and the analytical model predict a few unexpected behaviors.

II. Simulation

Using the OOMMF micromagnetic software package [1], we examined domain wall motion in a strip 5 nm thick and 1250 nm long. Our simulations included strips of width W ranging from 5 nm to 35 nm. Material parameters approximating Permalloy were chosen, saturation magnetization $M_S = 800$ kA/m and exchange energy coefficient $A = 13$ pJ/m. Crystalline anisotropy was not included in the simulation of this soft material. The OOMMF software computes magnetization dynamics according to the Landau-Lifshitz equation:

$$\frac{dm}{dt} = \frac{\gamma}{1+\alpha^2} H_{\text{eff}} \times m - \frac{\alpha\gamma}{1+\alpha^2} m \times H_{\text{eff}} \times m, \qquad (1)$$

where $\gamma = -221$ kHz/(A/m) is the gyromagnetic constant, α is a dimensionless phenomenological damping parameter, $m = M/M_S$ is normalized magnetization, and H_{eff} is the effective field representing the effect of all energies included in the simulation.

¿From a prior simulation study of static domain walls in thin, narrow strips, we expect head-to-head domains to be separated by a transverse domain wall [2]. As in that study, we are interested in behavior of the domain wall down the length of the strip, and not its interactions with the ends. To suppress edge effects, we calculate the magnetostatic stray field produced by magnetic charges of magnitude M_S and opposite in sign to the charges present in the head-to-head domain configuration on the ends of the strip. During the simulation, we apply these calculated fields. The effect is to remove the influence of the strip edges on the domain wall at a distance to approximate the simulation of domain wall motion along an infinitely long strip. Another useful artifact of this technique is that the domain wall is repelled from the strip ends by these artificial charges, keeping the wall from running completely off the simulated element. This is especially helpful in establishing a stable initial wall.

The initial transverse domain wall configuration is established in the element, with the domain wall near the left end of the strip, and allowed to relax to a stable state in which the domain wall remains near the left end. A field pulse is applied along the strip axis,

$$\mu_0 H_x(t) = \mu_0 H_{\text{app}}(1 - \cos 2\pi f t), 0 < t < 1\text{ns} \qquad (2)$$

where $f = 2$ GHz, so that the 1 ns pulse includes one full cosine period. Pulse magnitudes $\mu_0 H_{\text{app}}$ from 1 to 10 mT were applied. In response to each applied field pulse, we observe the transverse domain wall moving to the right. After the pulse has ended, the domain wall continues to move with a momentum of its own. Simulations with $\alpha = 0$ demonstrate that the domain wall motion is primarily a precessional effect.

By observation, we see that the transverse domain wall holds its shape throughout the simulation, and the domains remain uniformly magnetized along the strip axis, so the velocity of the wall can be simply derived from the average x component of magnetization of the whole element,

$$v(t) = \frac{L}{2} \frac{d < m_x(t) >}{dt}. \qquad (3)$$

With no damping, the domain wall momentum continues to move the wall at constant velocity. When damping is included, the domain wall comes to a stop some time after the pulse ends, although for large enough applied field pulse magnitudes, the wall velocity is observed to accelerate after the pulse ends before slowing to a stop.

Figure 1 graphs constant wall velocities observed in the zero-damping case for several values of strip width W and pulse magnitude $\mu_0 H_{\text{app}}$. For each W, there is a pulse magnitude that maximizes wall velocity. For wider strips, a lesser pulse magnitude produces the maximum velocity and that maximum velocity is greater.

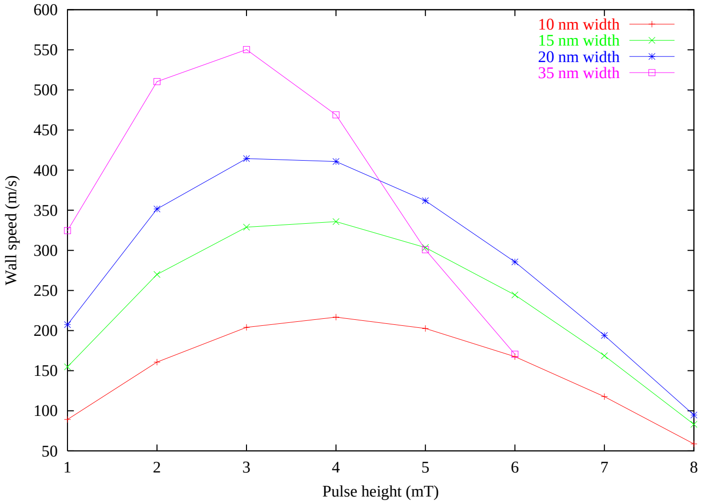

Fig. 1. Domain wall velocity for various strip widths and applied field pulse magnitudes

Another set of simulations was performed using an applied field step rather than an applied field pulse. The remarkable observation was that for large enough applied fields in the positive x direction, the domain wall velocity becomes negative part of the time, leading to a retrograde motion of the domain wall.

III. Domain Wall Structure

As a first step toward deriving an analytical model to explain these simulation results, we examine the structure of the domain wall itself. First we note that the exchange energy enforces a smoothness of spatial variation of magnetization. Also, the thickness and width of the strip are small compared to the spatial rate of change in magnetization, so to a good approximation, magnetization can be considered uniform in those dimensions. That is, the magnetization can be considered to vary only along the x axis, $M(x, y, z) = M(x)$.

There are two competing energies that give rise to the domain wall. Exchange energy prefers to spread the wall along the entire length of the strip. The shape anisotropy of the strip, however, prefers magnetization aligned with the strip axis, which tends to expand the domains at the expense of the wall. The actual width of the domain wall comes from a balancing of these two energies, in a manner precisely analogous the the well-known one-dimensional domain wall model where exchange and crystalline anisotropy energies are balanced.

Most of the shape anisotropy energy comes from charges on the edges of the strip due to the transverse component of magnetization in the domain wall. As an approximation, we neglect other sources of magnetostatic energy (including bulk charges), and computed the demagnetization energy as

$$E = -\frac{\mu_0 M_S}{2} \int_V m_y(x) H_y(x, y, z) dx dy dz, \tag{4}$$

where $H_y(x)$ arising from the surface charges is

$$\begin{aligned} H_y(x, y, z) &= \textstyle\int_0^L \int_0^T m_y(x') \quad M_S\{f(x - x', y - W, z - z') \\ &\qquad - f(x - x', y, z - z')\} dz' dx', \end{aligned}$$

where

$$f(x, y, z) = \frac{y}{[x^2 + y^2 + z^2]^{\frac{3}{2}}}. \tag{5}$$

After rearrangement and simplification

$$E = \frac{\mu_0 M_S^2}{2} \int_0^L \int_0^L m_y(x) m_y(x') \Phi(x - x') dx dx', \tag{6}$$

where Φ is integrable and has most weight near zero, so acts approximately as a Dirac delta function. Following the same calculus of variations analysis as for crystalline anisotropy energy,

$$E = K \int_0^L m_y^2(x) dx, \tag{7}$$

leads to the expression for an effective shape anisotropy constant for a given strip width W and thickness T,

$$\begin{aligned} K_m &= \tfrac{\mu_0 M_S^2}{2}\{ \quad 1 - \frac{2}{\pi} \tan^{-1}(\frac{W}{T}) \\ &\qquad + \frac{1}{2\pi} \frac{T}{W} \log(1 + (\frac{W}{T})^2) \\ &\qquad - \frac{1}{2\pi} \frac{W}{T} \log(1 + (\frac{T}{W})^2)\} \end{aligned} \tag{8}$$

Following the classical analysis of one-dimensional models of domain walls, this approximation predicts the domain wall width to be

$$a = \pi \sqrt{\frac{A}{K_m}}. \tag{9}$$

¿From the simulations, we can compute a different estimate of the domain wall width of the magnetization state,

$$\hat{a} = L(< m_y >^2 + < m_z >^2)^{\frac{1}{2}}. \tag{10}$$

The $\hat{a}$ estimate from simulations are consistently slightly larger (10 - 20 %) than the predicted value a, presumably due to the neglecting of bulk charges.

IV. Analytical Model

Having determined the expected domain wall width, we use it to construct a simplified view of the moving domain wall. Consider a partition of the strip into three regions: the two domains, each uniformly magnetized, and the domain wall uniformly magnetized in the transverse direction over a length a of the strip. The magnetization in the domains is parallel to the applied field pulse, so they do not react to it, either in precession or in damping. The domain wall region does respond. Damping toward the applied field causes the domain wall to rotate toward the positive x axis. Precession about the applied field causes the domain wall magnetization to tilt out of the plane of the strip at an angle θ. After the magnetization tilts out of plane, it is no longer anti-parallel to the demagnetization field. The component of demagnetization field perpendicular to the magnetization, $H_D^{\perp}$ is

$$H_D^{\perp} = M_S(N_z - N_y) \cos\theta \sin\theta, \tag{11}$$

where N_y and N_z are the demagnetizing factors of the $a \times W \times T$ region containing the domain wall [3]. For non-zero θ, $H_D^{\perp}$ is also non-zero, and the domain wall magnetization will precess around it, contributing to the rotation toward the positive x axis. The complete expression for velocity of the domain wall predicted by this simple model is

$$v = (\gamma/\pi)(H_D^{\perp} + \alpha H_{\mathrm{app}})a, \tag{12}$$

After the applied field pulse returns to zero, it is the precession about the demagnetizing field that sustains the momentum of the domain wall motion. This phenomenon of domain wall momentum is completely analogous to the momentum predicted by a one-dimensional model of a Bloch wall [4]. In both cases it is precession about a local demagnetization field that sustains wall motion. Damping will slowly draw energy from the system, and eventually bring the domain wall to a stop.

It is clear from these expressions that for any particular strip geometry, there is a tilt angle θ that maximizes domain wall velocity. The time rate of change of the tilt angle is

$$\theta' = \gamma(H_{\mathrm{app}} - \alpha H_D^{\perp}). \tag{13}$$

For the applied field pulses, the total tilt angle θ achieved by the end of the pulse is proportional to the area under the applied field pulse. It is also clear that larger velocities are expected as $(N_z - N_y)$ grows larger; that is, as the width-to-thickness ratio of the strip increases. These relationships explain the features of the micromagnetic simulation results in Fig. 1. When the applied field pulse creates a tilt angle θ greater than that which maximizes velocity, the model predicts that after the pulse, as damping decreases θ, the wall velocity will actually increase before it decreases and the wall comes to a stop, just as observed in micromagnetic simulation.

This analytical model can also explain the response of domain walls to a constant applied field. When the applied field is small enough, its tendency to increase the tilt angle θ will eventually be exactly balanced by the tendency of the damping to push θ back to zero. Specifically, for $H_{\mathrm{app}} < \alpha M_S(N_z - N_y)/2$, a constant θ is reached and the wall moves at the constant velocity determined by that tilt angle.

to retrograde domain wall motion observed in response

For larger H_{app}, θ will continue to grow as precession about the applied field continues past the z axis ($\theta = \pi/2$). Once θ exceeds $\pi/2$, both precession and damping combine to accelerate the magnetization back into the plane. Though precession continues clockwise around $H_D^{\perp}$, the transverse direction of magnetization is reversed, so that precession moves the wall in the reverse direction. That is, the domain wall velocity becomes negative. As θ exceeds π, the magnetization passes through the plane of the strip, and the direction of $H_D^{\perp}$ is reversed, causing the precession direction to reverse, yielding another reversal of wall direction. The same pattern repeats as the precession about the applied field continues for $\pi < \theta < 2\pi$. The number of cycles of domain wall direction reversal is exactly twice the number of precession rotations about the applied field.

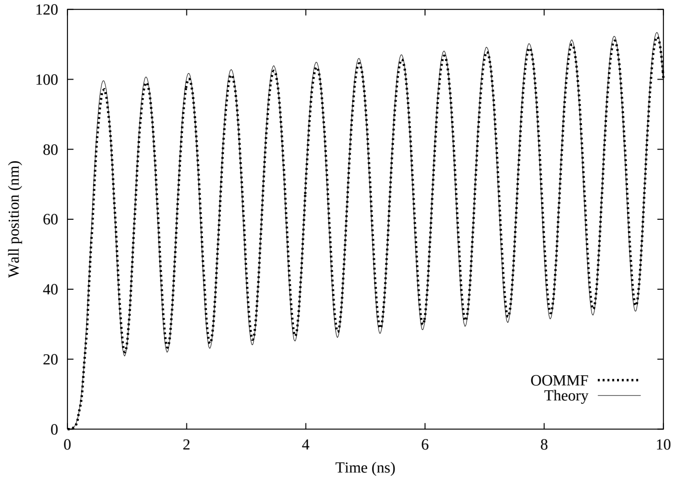

Fig. 2. Comparison of the predictions of the analytical model with the results computed by a full micromagnetic simulation. Response of a transverse domain wall to an applied field ramped up to a constant value. $\mu_0 H_{\mathrm{app}} = 25$ mT, $W = 15$ nm, $\alpha = 0.001$.

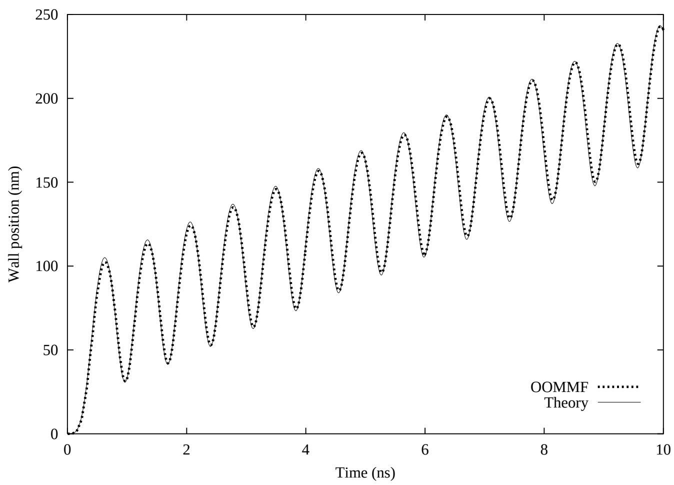

Fig. 3. Comparison of the predictions of the analytical model with the results computed by a full micromagnetic simulation. Response of a transverse domain wall to an applied field ramped up to a constant value. $\mu_0 H_{\mathrm{app}} = 25$ mT, $W = 15$ nm, $\alpha = 0.01$.

Figures 2 and 3 depict how well the simple analytical model succeeds in predicting the same domain wall position as a function of time as a full micromagnetic calculation.

References

[1] Michael J. Donahue and Donald Gene Porter, "OOMMF user's guide, version 1.0", NISTIR 6376, National Institute of Standards and Technology, 100 Bureau Drive, Gaithersburg, MD 20899, 1999.

[2] Robert D. McMichael and Michael J. Donahue, "Head to head domain wall structures in thin magnetic strips", *IEEE Trans. Magn.*, vol. 33, pp. 4167–4169, 1997.

[3] Andrew J. Newell, Wyn Williams, and David J. Dunlop, "A generalization of the demagnetizing tensor for nonuniform magnetization", *Journal of Geophysical Research - Solid Earth*, vol. 98, no. B6, pp. 9551–9555, June 1993.

[4] Sushin Chikazumi, *Physics of Magnetism*, p. 348, Robert E. Krieger Publishing Company, 1964.